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INCORPORATING FIXED-POINT AND -LINE CONSTRAINTS AND TOLERANCE BASED SYNTHESIS IN 4MDS

Anurag Purwar, Saurabh Bhapkar, Q.J. Ge

anurag.purwar@stonybrook.edu
Computational Design Kinematics Lab
Department of Mechanical Engineering
Stony Brook University
Stony Brook, New York, 11794-2300

ABSTRACT

*This paper presents implementation of fixed-pivots, ground-link line, and tolerance based motion synthesis in the 4MDS (**F**our-Bar **M**otion **D**esign **S**ystem). This is a continuation of the first work reported on 4MDS, which provides an interactive, graphical, and geometric constraint based mechanism design system for the exact- and approximate-motion synthesis problems. Theoretical foundation of the 4MDS is laid over a kinematic mapping based unified formulation of the geometric constraints (circle, fixed-line, line-tangent-to-a-circle) associated with the mechanical dyads (RR, PR, and RP) of a planar four-bar mechanism. An efficient algorithm extracts the geometric constraints of a given motion task and determines the best dyad types as well as their dimensions that best fit to the motion. Often, Mechanism designers need to impose additional geometric constraints, such as specification of location of fixed pivots or ground-link line. If synthesized mechanism suffers from branch, circuit, or order defect, they may also desire rectified solutions by allowing a tolerance to certain or all task positions. Such functions are crucial to a practitioner and much needed during the conceptual design stage of machine design process.*

1 Introduction

This paper is a continuation of work reported earlier in Purwar et al. [1], wherein we presented a planar Four-Bar Motion Design System (4MDS). The 4MDS is a geometric constraint based motion generation system that synthesizes planar four-bar linkages of all types. The core algorithm in 4MDS is an implementation of the simultaneous type and dimensional synthesis approach presented in Ge and Purwar [2] and Ge et al. [3]. The algorithm extracts the geometric constraints hidden in a given motion task to determine the best linkage type consisting of revolute- or prismatic-joints and provides the dimensional parameters. On the simulation side, the 4MDS provides a natural environment for the mechanism designers to input a mechanism by assembling geometric constraints associated with mechanical dyads (circle, fixed-line, and line-tangent-to-circle), examine the circuits and branches, observe the effect of actuating dyads on branching, and simulate the mechanism. In Synthesis mode, the 4MDS is capable of synthesizing linkages for $N(\geq 5)$ task positions for both exact- and approximate- synthesis problems. For a given motion, the designer can select from up to 4 possible dyads of types *RR*, *RP*, and *PR* to assemble a total of up to 6 linkages.

However, to enable adoption among practitioners, it is important that the system has the capability to admit designers' constraints. Often, mechanisms found suffer from undesirable Circuit, Branch, or Order defects (Chase and Mirth [4]). One approach to solving this problem is to allow tolerance in the given task positions if the problem permits and thereby expand the solution space from which potentially defect-free mechanisms can be found. Venkataraman et al. [5] proposed this for generating

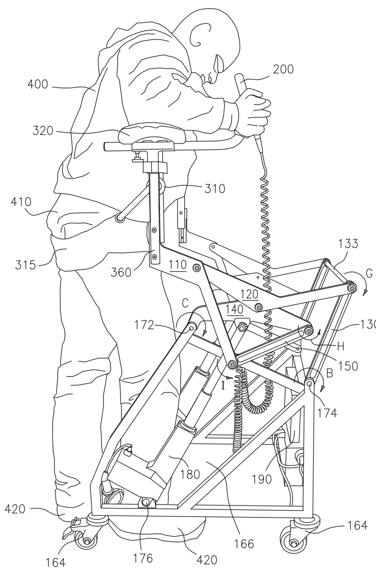

FIGURE 1. Example: A Sit-To-Stand Mobility Assist Device employs two identical planar linkages, one on each side to aid in getting up. Fixed pivots are constrained to be on a compact, portable frame.

four-bar linkages for four-position synthesis problem. Prof. Ken Waldron and his colleagues have authored many papers on the topic of branch- and circuit-defect rectification in planar linkages, a summary of which can be found in a recent publication by Chase et al. [6]. The new version of 4MDS takes this approach by allowing tolerances in the orientation part of the given positions. The number of new positions and range of rotation are completely controlled by the designer.

Apart from the aforementioned defect-rectification, the mechanisms may also need to satisfy additional geometric constraints, such as the location of fixed- or moving-pivots or the location of the ground-link line, etc. For example, a key requirement in a portable Sit-To-Stand Mobility Assist device that employs a linkage in assisting a disabled individual to execute a standing up motion is that it be compact and the positions of the fixed pivots be located on the frame; see Fig. 1. [1] The algorithm employed in 4MDS allows imposition of additional such geometric constraints. This paper implements an approach presented in Ge et al. [8], wherein we show that for four-positions synthesis problem, a ∞^1 solutions can be reduced to a finite number of solutions by assigning a line for the fixed pivots, while for three-position synthesis problems, which admit ∞^2 solutions, two separate fixed pivots can be specified. We review this approach in this paper.

Apart from the technical capabilities of 4MDS, hitherto unknown in any mechanism design and simulation software, the system provides an intuitive, fluid interface that facilitates a dialog with the designer. Simulation and synthesis features provide a lot of flexibility – for example, after synthesizing a mechanism, the mechanism can be edited and the designer can compare the new motion with the old one. Another example is that of approximating an RR dyad by an RP or PR dyad by restricting workspace size or limiting the length of links.

This body of work is placed under the topic of *Kinematic Synthesis of Mechanisms* covered extensively in books by McCarthy and Soh [9], Sandor and Erdman [10], Hunt [11], Hartenberg and Denavit [12], and Suh and Radcliffe [13]. The key papers related to this work use kinematic mapping approach (Blaschke [14] and Grunwald [15]). Readers are referred to Bottema and Roth [16], McCarthy [17] and Ravani and Roth [18,19] for a modern treatment and application of kinematic mapping. The problem itself, that is of motion generation for a set of given task positions is closely related to the classical Burmester problem [20], which has been attempted in the last decade to include both revolute and prismatic joints ([21, 22, 23, 24]). The development of 4MDS falls under the topic of *Computer Aided Mechanism Design*, a detailed review of which can be found in Chase et al. [6] and summarized in Purwar et al. [1], both published recently.

The 4MDS software developed indigenously at Stony Brook University is a work in progress and assumes no liability for its use. It can be downloaded at http://cadcam.eng.sunysb.edu/software or at http://me.eng.sunysb.edu/software.

The organization of the rest of the paper is as follows. Section 2 reviews kinematic fundamentals and a formulation of geometric constraints of dyads insofar as necessary for this paper. Section 3 presents tolerance based synthesis approach, while sections 4 and 5 present the incorporation of fixed ground-link and fixed-pivot constraints in 4MDS, respectively. Additional features and functionalities in 4MDS are presented in the latter three sections as appropriate.

2 Theoretical Formulation

The 4MDS's core functionality is rooted in an implementation of our task driven approach to unified type and dimensional synthesis of planar four-bar linkages (Ge and Purwar [2]). This paper reports a computational implementation of our work in [8] on incorporating additional geometric constraints, such as fixed pivots, ground-link line, and tolerance based synthesis in 4MDS. In [8], we have shown that additional geometric constraints can be handled in a simple way without changing the core algorithm. In what follows, we briefly review some of the fundamentals from these two papers insofar as necessary to demonstrate new functions in 4MDS. We refer the reader to the original papers for more details.

[1] In fact, the development of 4MDS was motivated by such practical concerns and difficulties faced by Purwar et al. [7] during design of this device.

2.1 Kinematic Mapping of Planar Displacements

Let M denote a coordinate frame attached to the moving body and F be a fixed reference frame. Then, a planar displacement given by translation (d_1, d_2) of a point and rotation ϕ in M can be parameterized by planar quaternion coordinates $\mathbf{Z} = (Z_1, Z_2, Z_3, Z_4)$ (Ravani and Roth [18] and McCarthy [17]) using the following kinematic mapping

$$\begin{aligned} Z_1 &= \frac{1}{2}(d_1 \sin\frac{\phi}{2} - d_2 \cos\frac{\phi}{2}), \\ Z_2 &= \frac{1}{2}(d_1 \cos\frac{\phi}{2} + d_2 \sin\frac{\phi}{2}), \\ Z_3 &= \sin\frac{\phi}{2},\ \ Z_4 = \cos\frac{\phi}{2}. \end{aligned} \tag{1}$$

Then, a planar displacement represented as a homogeneous transformation of point $\mathbf{x} = (x_1, x_2, x_3)$ or line $\mathbf{l} = (l_1, l_2, l_3)$ from M to F can be given by

$$\mathbf{X} = [H]\mathbf{x},\ \ [H] = \begin{bmatrix} Z_4^2 - Z_3^2 & -2Z_3Z_4 & 2(Z_1Z_3 + Z_2Z_4) \\ 2Z_3Z_4 & Z_4^2 - Z_3^2 & 2(Z_2Z_3 - Z_1Z_4) \\ 0 & 0 & Z_3^2 + Z_4^2 \end{bmatrix}, \tag{2}$$

$$\mathbf{L} = [\overline{H}]\mathbf{l},\ \ [\overline{H}] = \begin{bmatrix} Z_4^2 - Z_3^2 & -2Z_3Z_4 & 0 \\ 2Z_3Z_4 & Z_4^2 - Z_3^2 & 0 \\ 2(Z_1Z_3 - Z_2Z_4) & 2(Z_2Z_3 + Z_1Z_4) & Z_3^2 + Z_4^2 \end{bmatrix}, \tag{3}$$

where $Z_3^2 + Z_4^2 = 1$ and $\mathbf{X} = (X_1, X_2, X_3)$ and $\mathbf{L} = (L_1, L_2, L_3)$ are corresponding point and line coordinates in F.

2.2 Geometric Constraints of Mechanical Dyads

Mechanical dyads of types *RR*, *PR*, and *RP* in Planar four-bar linkages impose circle-, line-, or line-tangent-to-circle constraints on the end-effector, respectively. Using kinematic mapping, these geometric constraints can be written in a unifying algebraic form. Let $\mathbf{a} = (a_1, a_2, a_0)$, where $a_0 \neq 0$, denote homogeneous coordinates of the center of a circle C in F. Then a point with homogeneous coordinates $\mathbf{X} = (X_1, X_2, X_3)$ lies on C if

$$2a_1X_1 + 2a_2X_2 + a_3X_3 = a_0\left(\frac{X_1^2 + X_2^2}{X_3}\right). \tag{4}$$

The radius r of the circle is given by

$$r^2 = (a_1/a_0)^2 + (a_2/a_0)^2 + a_3/a_0. \tag{5}$$

When $a_0 = 0$, Eq.(4) becomes linear,

$$L_1X_1 + L_2X_2 + L_3X_3 = 0, \tag{6}$$

which represents a line with homogeneous coordinates $\mathbf{L} = (2a_1, 2a_2, a_3)$. Thus, Eq.(4) is a unified presentation for both a circle and a line, and therefore, could lead to a unified representation of the constraints of *RR* and *PR* dyads.

For an *RP* dyad, a line with homogeneous coordinates $\mathbf{L} = (L_1, L_2, L_3)$ passes through a fixed point $\mathbf{X} = (X_1, X_2, X_3)$. In other words, they also satisfy (6).

Thus, we may conclude that all three dyadal constraints can be represented by Eq. (4) and that when $a_0 = 0$, the dyad has at least one prismatic joint.

By substituting (2) into (4), we have shown in [2] that the constraint manifold of an *RR* dyad is the following quadric surface

$$\begin{aligned} &p_1(Z_1^2 + Z_2^2) + p_2(Z_1Z_3 - Z_2Z_4) + p_3(Z_2Z_3 + Z_1Z_4) \\ &\quad + p_4(Z_1Z_3 + Z_2Z_4) + p_5(Z_2Z_3 - Z_1Z_4) + p_6Z_3Z_4 \\ &\quad + p_7(Z_3^2 - Z_4^2) + p_8(Z_3^2 + Z_4^2) = 0, \end{aligned} \tag{7}$$

where the eight coefficients p_i are not independent but must satisfy two quadratic conditions

$$p_1p_6 + p_2p_5 - p_3p_4 = 0,\ \ 2p_1p_7 - p_2p_4 - p_3p_5 = 0. \tag{8}$$

This is because p_i are related to the geometric parameters of the dyad by

$$\begin{aligned} &p_1 = -a_0, \quad p_2 = a_0x \quad p_3 = a_0y, \quad p_4 = a_1, \quad p_5 = a_2, \\ &p_6 = -a_1y + a_2x, \quad p_7 = -(a_1x + a_2y)/2, \\ &p_8 = (a_3 - a_0(x^2 + y^2))/4, \end{aligned} \tag{9}$$

where (a_0, a_1, a_2, a_3) are the homogeneous coordinates of the constraint circle and (x, y) are the coordinates of the circle point. For a *PR* dyad, we have $a_0 = 0$ and therefore, $p_1 = p_2 = p_3 = 0$. Eqns. (7) and (8) are said to define the constraint manifold of *RR* and *PR* dyads.

By substituting (3) into (6), it is found that for *RP* dyad, the constraint manifold has the same form as Eqns. (7) and (8), however we now have $p_1 = p_4 = p_5 = 0$. Thus, all planar dyads can be represented in the same form by Eqns. (7) and (8), and we can determine the type of a planar dyad by looking at the zeros in the coefficients p_i (called signature of a dyad). In our approach, we first obtain the homogeneous coordinates p_i, determine the dyad type from the signature of coefficient array p_i, and then compute the dyad parameters using inverse relationships in (9).

3 Tolerance Based Synthesis

The synthesis of planar four-bar linkage in 4MDS is based on two fundamental steps: 1. fitting of given task image points to a pencil of quadrics (Eq. (7)), 2. imposing additional constraints (Eq. (8)) on the pencil to reveal constraint manifold of dyads (see [2] for details). Given N task positions ($N \geq 5$), substituting for their quaternion components in Eq. (7), N linear equations are obtained. However, since there are no more than five independent dyad parameters, the system of equation $[A]\mathbf{p} = 0$ is over-constrained, where $\mathbf{p}$ is the column vector of homogeneous coefficients $p_i (i = 1 \ldots 8)$. The coefficient matrix $[A]$ is given by:

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\ \vdots & & & & & & & \vdots \\ \vdots & & & & \ddots & & & \vdots \\ \vdots & & & & & & & \vdots \\ A_{N1} & A_{N2} & A_{N3} & A_{N4} & A_{N5} & A_{N6} & A_{N7} & A_{N8} \end{bmatrix} \quad (10)$$

where, for the ith image points, we have

$$\begin{aligned} A_{i1} &= Z_{i1}^2 + Z_{i2}^2, \quad A_{i2} = Z_{i1}Z_{i3} - Z_{i2}Z_{i4}, \\ A_{i3} &= Z_{i2}Z_{i3} + Z_{i1}Z_{i4}, \quad A_{i4} = Z_{i1}Z_{i3} + Z_{i2}Z_{i4}, \\ A_{i5} &= Z_{i2}Z_{i3} - Z_{i1}Z_{i4}, \quad A_{i6} = Z_{i3}Z_{i4}, \\ A_{i7} &= Z_{i3}^2 - Z_{i4}^2, \quad A_{i8} = Z_{i3}^2 + Z_{i4}^2. \end{aligned} \quad (11)$$

Using Singular Value Decomposition (SVD), the over-constrained system of linear equations can be solved as a total least squares minimization problem with the constraint $\mathbf{p}^T\mathbf{p} = 1$. The solutions are the right singular vectors of $[A]$ corresponding to the least singular values. These vectors form an orthonormal set of basis vectors spanning the nullspace of $[A]$. For $N \geq 5$, the rank of matrix $[A]$ is five, then the matrix $[A]^T[A]$ has three zero eigenvalues and the corresponding eigenvectors, $\mathbf{v}_\alpha$, $\mathbf{v}_\beta$ and $\mathbf{v}_\gamma$, define the basis for the null space. Let α, β, γ denote three real parameters. Then, any vector in the null space is given by

$$\mathbf{p} = \alpha\mathbf{v}_\alpha + \beta\mathbf{v}_\beta + \gamma\mathbf{v}_\gamma. \quad (12)$$

The above also defines a pencil of quadrics as α, β, and γ are varied.

For vector $\mathbf{p}$ to satisfy Eq. (8), we substitute (12) into (8) and obtain two homogeneous quadratic equations in (α, β, γ):

$$\begin{aligned} K_{10}\alpha^2 + K_{11}\beta^2 + K_{12}\alpha\beta + K_{13}\alpha\gamma + K_{14}\beta\gamma + K_{15}\gamma^2 = 0, \\ K_{20}\alpha^2 + K_{21}\beta^2 + K_{22}\alpha\beta + K_{23}\alpha\gamma + K_{24}\beta\gamma + K_{25}\gamma^2 = 0, \end{aligned} \quad (13)$$

where K_{ij} are defined by components of the three eigenvectors obtained from SVD algorithm. These two quadratic equations

FIGURE 3. Icons that govern task specification and editing; from left to right: home (reset), read input file, write output, input task position, edit task position, synthesize

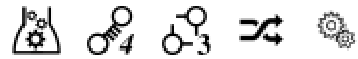

FIGURE 4. Synthesis options; from left to right: Tolerance Based Synthesis, Four Positions Synthesis, Three Positions Synthesis, Generate Arbitrary Positions, Settings

can be further reduced to a single quartic equation in one unknown parameter in terms of the ratio of two of the three parameters (α, β, γ) and thus can be analytically solved.

For given N positions, if either of the four possible dyad solutions do not satisfy the designer's requirements, such as a circuit, branch, or order defect-free mechanism, the 4MDS allows the designer to specify a tolerance for all the given positions so as to expand the solution space. In the current implementation, the designer can vary the orientation part of given positions. This allows a simultaneous change in all of the four quaternion parameters. For example, generating 3 new orientations in the vicinity of the original orientation for each of the 5 positions in a given motion computes up to 4 dyads for each of the new 243 task sets in less than 5 seconds. We illustrate this via an example for five task positions given in Table 1.

TABLE 1.

d_1	d_2	ϕ (degree)
-5.58	1.09	43.8
-2.92	1.84	11.4
-5.29	1.73	-16.26
1.14	5.04	-57.33
2.32	-1.98	-118.8

1. Select *Input Task* to enter five positions interactively or from a file (see Fig. 3 for icon details)
2. After positions are entered, select *Tolerance Based Synthesis* icon (see Fig. 4).
3. In the *Tolerance Specification* dialog, select Tolerance value (15^0 in this case) and number of positions (3 in this case); this creates three new positions: $-15^0, 0, +15^0$.

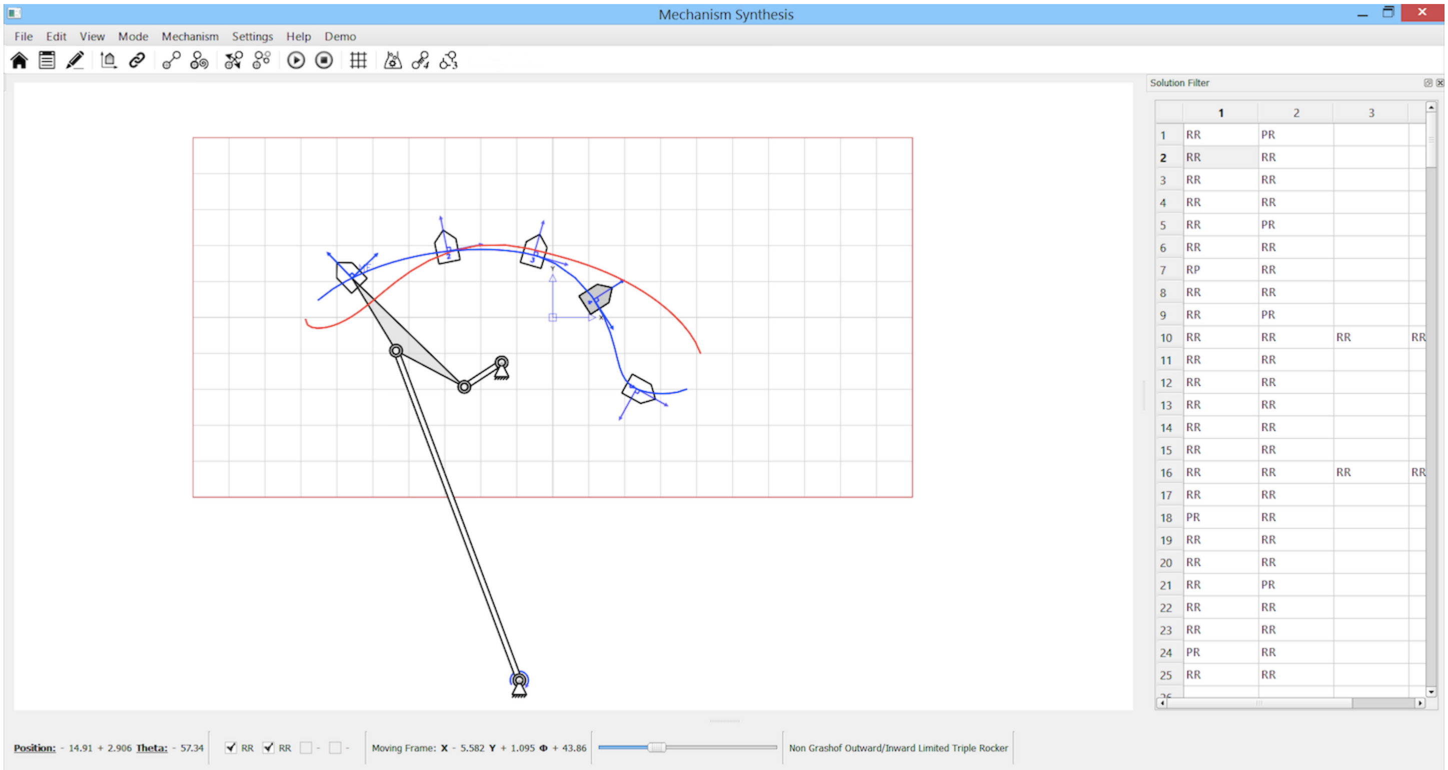

FIGURE 2. A screenshot of the 4MDS showing Design Space of the synthesized five toleranced positions.

4. Based on the Tolerance specification, a number of solutions are computed ($3^5 = 243$ in this case) and stored in a tabular form as shown in Fig. 2.
5. Click on a solution in the table to load dyads. Choose two dyads at a time at the bottom of the program window. The shown solution is circuit-, branch- and order-defect free.
6. Save dyad parameters data; this data is automatically saved in a file *dyad parameters.txt*. Table 2 contains the computed dyad parameters data.

TABLE 2.

Dyad Type	a_1/a_0	a_2/a_0	x_1/x_3	x_2/x_3	r
RR	-0.92	-10.09	-0.51	-2.3	9.78
RR	-1.41	-1.25	0.15	-4.34	1.24

We are implementing a user-defined filter that would automatically sort the produced linkages based on user's preferences thereby removing the tedium of manually inspecting solutions.

Comment on Synthesis and Simulation

After the algorithm has computed dyad solutions, the designer can choose two dyads (see Fig. 5) to display a four-bar linkage. The determination of the dyad type is based on practical constraints of workspace size. Instead of directly determining the dyad type from the signature of the $\mathbf{p}-$ vector, the dyads are interpreted to be an RP or PR from an RR dyad if its radius is larger than a multiple of the workspace size. If an RR dyad is found to be longer than the selected maximum radius, it is forced to be interpreted as either a RP or a PR dyad based on the location of the fixed pivot. For an RP dyad, the approximating RR dyad has a large radius, but the center is not far off, while for a PR dyad, the radius is large as well as the center is far off. All of these multiples are completely user-customizable – the user can change the workspace size and also the maximum link length (radius) as a multiple of the workspace width in the *Settings* option. In the dyad selection widget, the PR dyad in Fig. 5 has been reinterpreted from an RR dyad – the reinterpreted dyad is shown with a $*$ next to it. The user can always see the original RR dyad and even change the workspace size and multiple factor to see how this interpretation changes the dyad type. Choosing a very low multiple factor may yield poor approximation to the given motion, while a very large factor may yield impractically large mechanisms. A dyad obtained from synthesis or entered interac-

FIGURE 5. Checkboxes enable designer to select two dyads at a time. The one marked with an asterisk is a re-interpreted dyad based on workspace size and the maximum link length.

FIGURE 6. These icons allow designer to input dyadal constraints interactively; from left to right: assemble a mechanism mode, edit assembled mechanism mode, circle (RR-dyad) constraint, fixed-line (PR-dyad) constraint, moving-line (RP-dyad) constrain, moving frame location

tively can be edited by selecting Mechanism Edit Mode (Fig. 6) and then dragging the pivots or clicking on a dyad to provide numerical data. The existing dimensions of the dyad are displayed in the bottom left corner and can be edited.

4 Fixed Ground-link Line Constraint

It is well-known that for 4 position motion synthesis problems, there are ∞^1 linkage solutions. A simple way to reduce them to finite solutions is to impose a constraint on the locations of the fixed pivot, such as that they lie on a given fixed line. In this case, the matrix $[A]$ in (10) becomes a 4×8 matrix and thus the matrix $[A]^T[A]$ is of rank 4. The application of SVD algorithm yields four zero eigenvalues. Let $\mathbf{v}_\alpha$, $\mathbf{v}_\beta$, $\mathbf{v}_\gamma$ and $\mathbf{v}_\mu$ denote four eigenvectors associated with the zero eigenvalues. They form the basis of the four-dimensional null space of $[A]$. Any vector $\mathbf{p}$ in the null space as given by

$$\mathbf{p} = \alpha \mathbf{v}_\alpha + \beta \mathbf{v}_\beta + \gamma \mathbf{v}_\gamma + \mu \mathbf{v}_\mu \tag{14}$$

defines a candidate constraint manifold of a planar dyad that is compatible with the four specified task positions. We need to select the real parameters $(\alpha, \beta, \gamma, \mu)$ such that both constraints in Eqs.(8) are satisfied. This leads to

$$\begin{aligned} K_{10}\alpha^2 + K_{11}\beta^2 + K_{12}\mu^2 + K_{13}\alpha\beta + K_{14}\alpha\mu + K_{15}\beta\mu + K_{16}\alpha\gamma \\ + K_{17}\beta\gamma + K_{18}\mu\gamma + K_{19}\gamma^2 = 0, \\ K_{20}\alpha^2 + K_{21}\beta^2 + K_{22}\mu^2 + K_{23}\alpha\beta + K_{24}\alpha\mu + K_{25}\beta\mu + K_{26}\alpha\gamma \\ + K_{27}\beta\gamma + K_{28}\mu\gamma + K_{29}\gamma^2 = 0, \end{aligned} \tag{15}$$

where K_{ij} are obtained from $\mathbf{Z}_i$ $(i = 1, 2, 3, 4)$. Thus, the homogeneous parameters $(\alpha, \beta, \gamma, \mu)$ have ∞^1 many solutions .

An easy way to obtain finite solutions without increasing the complexity of the problem is to impose a linear relationship among these parameters, i.e.,

$$k_1\alpha + k_2\beta + k_3\gamma + k_4\mu = 0. \tag{16}$$

By varying the choice of k_i, one obtains different solutions for $(\alpha, \beta, \gamma, \mu)$.

Constraining the center point (X_c, Y_c) of an RR dyad to a line $L_1X_c + L_2Y_c + L_3 = 0$, we obtain one form of Eq. (16) as

$$-L_1p_4 - L_2p_5 + L_3p_1 = 0, \tag{17}$$

where p_1, p_4, p_5 are functions of $(\alpha, \beta, \gamma, \mu)$ via Eq. (14). Let us illustrate this via an example for four task positions and fixed line coordinates given in Table 3:

TABLE 3.

d_1	d_2	ϕ (degree)
-6.89	3.80	-4
1.80	3.23	-33.6
3.71	1.81	-58.9
4.92	-7.6	-77.1
L_1	L_2	L_3
-0.4281	1	1.5658

1. Select *Input Task* and enter four positions interactively from the Table 3 or from a file
2. After four positions are entered, select *Four Positions* icon (see Fig. 4). Interactively draw a line that represents the fixed ground-link line constraint. Table 3 gives homogenous co-ordinates of this line.
3. As soon as the line is entered, dyads are computed. Select two dyads from the check boxes in the bottom row.
4. Swap actuating dyad, switch between open and crossed configuration and run simulation to observe the mechanism behavior: examine circuit, position traversal order, and type of the computed mechanism
5. Save dyad parameters data; this data is automatically saved in a file *dyad parameters.txt*. Table 4 contains the computed dyad parameters data.

Figure 7 shows the final synthesized mechanism.

5 Fixed Ground-Pivot Constraints

For three positions synthesis problem, which has ∞^2 number of solutions, imposing two linear constraints reduces the number

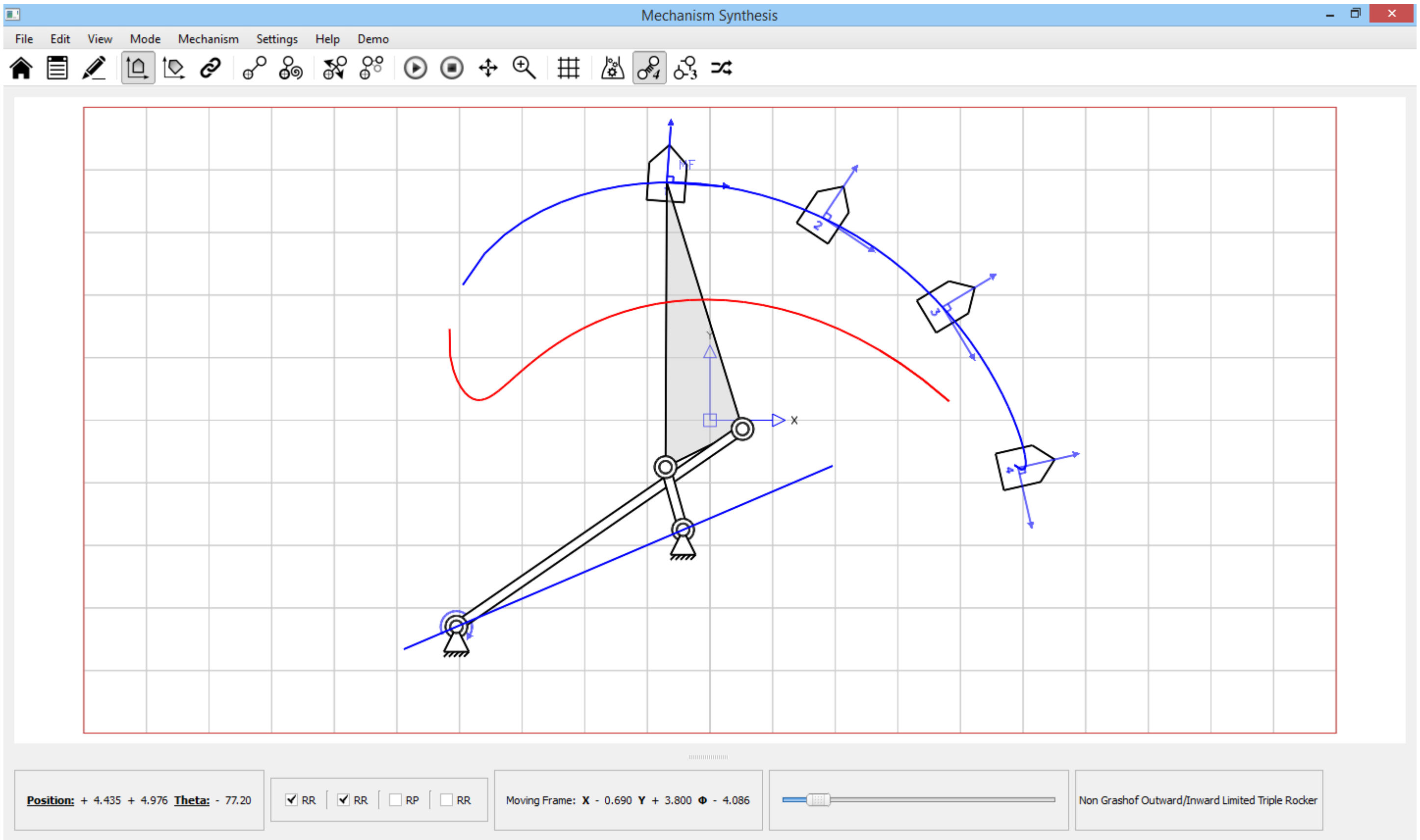

FIGURE 7. A screenshot of the 4MDS showing Design Space of the synthesized four positions with ground-link line constraint.

TABLE 4.

Dyad Type	a_1/a_0	a_2/a_0	x_1/x_3	x_2/x_3	r
RR	-4.05	-3.3	1.48	-3.84	5.55
RR	-0.43	-1.75	0.30	-4.54	1.03

TABLE 5.

d_1	d_2	ϕ (degree)
-0.43	2.77	-23.31
1.83	3.50	-72.59
3.06	0.14	-90
Pivot Points	X_c	Y_c
1	-5	1
2	0	-1

of solutions. In this case, there are only three linear equations in the form of (7) and the null space of the resulting coefficient matrix $[A]$ is five dimensional. There are five zero eigenvalues from the matrix $[A]^T[A]$. The corresponding eigenvectors are denoted by $\mathbf{v}_\alpha, \mathbf{v}_\beta, \mathbf{v}_\gamma, \mathbf{v}_\mu, \mathbf{v}_\eta$. A vector in the null space is given by

$$\mathbf{p} = \alpha\mathbf{v}_\alpha + \beta\mathbf{v}_\beta + \gamma\mathbf{v}_\gamma + \mu\mathbf{v}_\mu + \eta\mathbf{v}_\eta \tag{18}$$

and only those $\mathbf{p}$ that satisfy (8) define the constraint manifolds of feasible dyads. We may use two linear equations of the form (16) to limit solutions. We illustrate this via another example of 3 positions with fixed ground pivots given in Table 5.

1. Select *Input Task* icon and Enter three positions interactively or from a file.
2. After three positions are entered, select *Three Positions* icon (see Fig. 4). Interactively enter two points representing the location of the fixed pivots.
3. As soon as the points are entered, dyads are computed. Select two dyads from the check boxes in the bottom row.
4. Swap actuating dyad, switch between open and crossed configuration and run simulation to observe the mechanism behavior: examine circuit, position traversal order, and type of

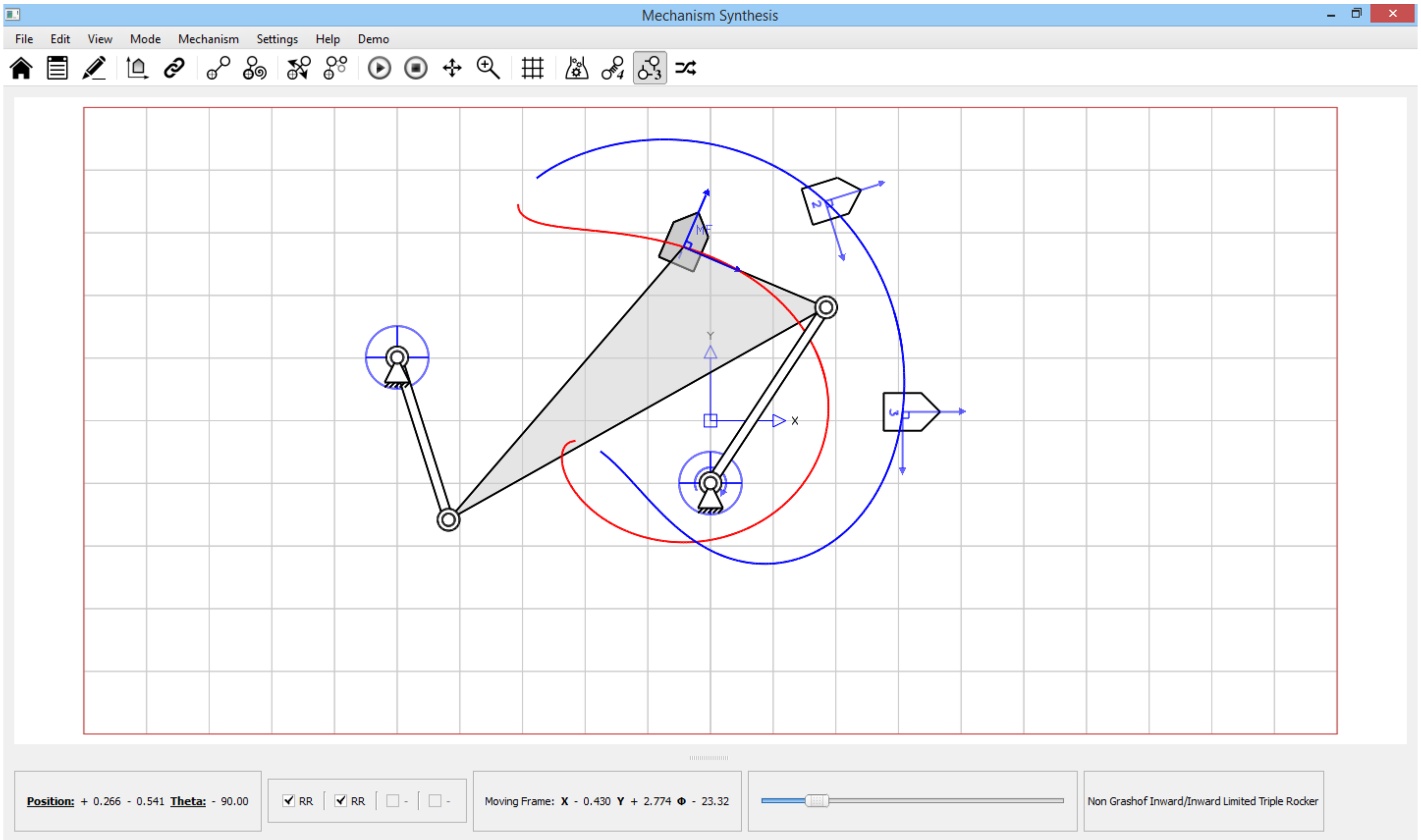

FIGURE 8. A screenshot of the 4MDS showing Design Space of the synthesized three positions with given ground pivots.

the computed mechanism

5. Save dyad parameters data; this data is automatically saved in a file *dyad parameters.txt*. Table 6 contains the computed dyad parameters data.

TABLE 6.

Dyad Type	a_1/a_0	a_2/a_0	x_1/x_3	x_2/x_3	r
RR	0	-0.99	2.47	0.01	3.35
RR	-4.99	1.01	-1.71	-5.48	2.71

Figure 8 shows the final synthesized mechanism.

6 Conclusions and Future Work

This paper presented an implementation of our recent work on incorporating practical geometrical constraints and tolerance based synthesis to a new interactive, GUI-driven planar four bar mechanism software system called 4MDS, which can compute both type and dimensions simultaneously for a given motion task. Such functions are necessary in a practical tool of its ilk. However, work remains on allowing designer to input other kinematic- and geometric-constraints, automated evaluation and filtering of computed linkages based on criteria such as circuit/branch/order defect, transmission angle for tolerance-based synthesis or incorporating the defect resolution in the synthesis algorithm.

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